

# Equation for one-loop divergences in two dimensions and its application to higher spin fields.

H.P.Popova

*Skobeltsyn Institute of Nuclear Physics MSU,  
space science division,  
119234, Moscow, Russia,*

K.V.Stepanyantz

*Moscow State University,  
physical faculty, department of theoretical physics,  
119991, Moscow, Russia*

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## Abstract

A simple formula for one-loop logarithmic divergences on the background of a two-dimensional curved space-time is derived for theories for which the second variation of the action is a nonminimal second order operator with small nonminimal terms. In particular, this formula allows to calculate terms which are integrals of total derivatives. As an application of the result, one-loop divergences for the higher spin fields on the constant curvature background are obtained in a nonminimal gauge, which depends on two parameters. By an explicit calculation we demonstrate that with the considered accuracy the result is gauge independent. Moreover, the result appeared to be independent of the spin  $s$  for  $s \geq 3$ .

Keywords: one-loop divergences, higher spin fields.

## 1 Introduction

Obtaining one-loop divergences is a typical calculational problem in quantum field theory. A standard method to solve it is to construct all divergent diagrams and to calculate them. However, this can lead to certain difficulties in some cases. For example, if the calculations are made in the curved space, a number of divergent diagrams is infinite in the case of constructing the perturbation theory on the flat background. Nevertheless, writing the result in the covariant form allows to find a sum of all divergent contributions in this case. The covariant result can be constructed, for example, by using the calculations in the weak field limit in which the deviation of the metric from the flat one is considered to be small or by some other methods. However, the calculations can be considerably simplified if some operations are made in the general case. This is done, for example, within the method for calculating one-loop divergences proposed by G.t'Hooft and M.Veltman [1], by which the one-loop divergences was first calculated for gravity.

The essence of this method is that if the second variation of the action (which determines the one-loop divergences) is the minimal second order operator, then it is possible to construct a covariant equation relating one-loop divergences to the coefficients of this operator. Then it is not already necessary to restore a covariant result from the weak field expansion in each

particular case. The t'Hooft–Veltman method allows to considerably simplify calculations of the one-loop divergences and to find easily their signs and coefficients. The main shortcoming of this method is strong limitations on a form of the second variation of the action. Using various techniques some generalizations of this method were constructed. In particular, the t'Hooft–Veltman formula has been generalized to the case of an arbitrary differential operator [2]. Although the result appeared to be very large, it has been applied for making some one-loop calculations [3, 4, 5, 6, 7]. Moreover, the obtained general formula allows to simplify automatization of the calculations, if the software for treating tensors (e.g., [8, 9]) is used.

From the mathematical point of view the t'Hooft–Veltman method corresponds to calculating the Minakshisundaram–Seeley–de-Witt coefficient [10, 11, 12, 13]  $b_4$  for a differential operator coinciding with the second variation of the action. In four dimensions this coefficient is related to one-loop logarithmic divergences. In the general case, in the space of the dimension  $d = 2n$  the one-loop logarithmic divergences are related to the coefficient  $b_{2n}$ .

For various differential operators the Minakshisundaram–Seeley–de-Witt coefficients can be calculated using the Schwinger–de-Witt technique and its generalizations [14, 15, 16, 17]. However, this technique does not allow to obtain a result for an arbitrary differential operator. From the other side, the total derivative terms were not found in Ref. [2]. (Their contributions to the one-loop divergences are integrals of total derivatives.) Therefore, there is a problem how the results of Ref. [2] can be generalized in order to take into account total derivative terms. These contributions are essential in some cases, for example, if the calculations are made on the (anti) de-Sitter background. Let us note that the calculations on the (anti) de-Sitter background are usually made by different methods. For example, the Minakshisundaram–Seeley–de-Witt coefficients for the minimal operator can be found in an arbitrary dimension by the harmonic analysis on the homogeneous spaces [18]. Using this method one-loop divergences were calculated for the fields of an arbitrary spin on the four-dimensional (anti) de-Sitter background in the minimal gauge [19]. A similar calculation, based on the formula for the  $b_4$  coefficient of the minimal operator, was made in Ref. [20].

In the present paper we try to understand, how it is possible to calculate total derivatives terms using the t'Hooft–Veltman technique for nonminimal operators. For this purpose we consider the simplest case of the two-dimensional space, in which logarithmic divergences are related to the coefficient  $b_2$ . We calculate this coefficient for the second order nonminimal operator taking into consideration the total derivative terms, under the assumption that nonminimal terms are small, but nonvanishing. Then this result is verified by a calculation of one-loop divergences for the higher spin theory in a nonminimal gauge, which depends on two arbitrary (small) parameters.

The paper is organized as follows. In Sect. 2 we describe a method for calculating one-loop divergences which is applicable, if the second variation of the action is a second order nonminimal operator with small nonminimal terms, and present the result of this calculation, which is given by Eq. (16). Using this result in Sect. 3 a divergent part of the one-loop effective action for the higher spin theory on the two-dimensional (anti) de-Sitter background is calculated in a nonminimal gauge depending on two parameters. For this purpose in Sect. 3.1 we recall the basic information about the higher spin theory on the constant curvature background. The calculation of one-loop divergences is described in Sect. 3.2. The result is given by Eq. (40). The results are briefly discussed in the conclusion. One technical problem related to the one-loop calculation is considered in the Appendix.

## 2 One-loop divergences in two dimensions for second order non-minimal operator

In the one-loop approximation the effective action is written in the form (see, e.g., [21])

$$\Gamma[\varphi] = S[\varphi] + \frac{i}{2}\hbar \text{Tr} \ln D + O(\hbar^2), \quad (1)$$

where the operation  $\text{Tr}$  by definition includes  $\int d^d x$ , and the differential operator  $D$  is the second variation of the classical action:

$$D_i^j = \frac{\delta^2 S}{\delta \varphi^i \delta \varphi_j}, \quad (2)$$

where each of the indexes  $i$  and  $j$  denotes the whole set of possible indexes of the fields  $\varphi$ . Most models considered in field theory are quadratic in the derivatives of fields. Therefore, for a large number of practical problems it is sufficient to consider only differential operators of the second order. Note that in some cases differential operators of higher orders are also interesting, see, for example, Ref. [22]. An arbitrary nonminimal differential operator of the second order has the form

$$D_i^j = (I g^{\mu\nu} \nabla_\mu \nabla_\nu + K^{\mu\nu} \nabla_\mu \nabla_\nu + S^\mu \nabla_\mu + W)_i^j, \quad (3)$$

where for later convenience we extract the terms containing the Laplace operator, and the covariant derivative is written as

$$(\nabla_\mu)_i^j = \delta_i^j \partial_\mu + \omega_{\mu i}^j, \quad (4)$$

where  $\omega_{\mu i}^j$  is a connection. The coefficients  $I_i^j$ ,  $K^{\mu\nu}_i{}^j$ ,  $S^\mu_i{}^j$ , and  $W_i^j$  are functions of the fields and are obtained by calculating the second variation of the action. Without loss of generality  $K^{\mu\nu}_i{}^j$  can be assumed to be symmetric in the indexes  $\mu$  and  $\nu$  that will be always assumed below. The term “nonminimal” means that terms with a maximal number of the derivatives do not coincide with the Laplace operator in a certain degree. (In the considered case these are the terms with the second derivatives, which should be compared with the first degree of the Laplace operator.)

We will calculate one-loop divergences using the dimensional regularization [23, 24, 25, 26]. The space-time dimension we will denote by  $d$ . If a theory is considered in two dimensions, then the divergent part of the one-loop effective action is proportional to  $(d-2)^{-1}$ , where  $d \rightarrow 2$  after the renormalization. Using the general coordinate invariance, it is convenient to present the divergent part of the one-loop effective action in the form

$$\Gamma_{1-loop}^{(\infty)} = \frac{1}{4\pi(d-2)} \int d^2 x \sqrt{-g} b_2, \quad (5)$$

where  $b_2$  is a covariant function of the fields. This function is the second Minakshisundaram–Seeley–de-Witt coefficient. In this paper we calculate it for the second order nonminimal differential operator, assuming that the nonminimal terms are small. Moreover, we will assume that  $(S^\mu)_i^j = 0$  that is valid for a large number of practical problems.

In order to find one-loop divergences we will use the generalization of the method proposed by G. 't Hooft and M. Veltman [1], using which the coefficient  $b_4$  was calculated for an arbitrary differential operator without taking into account the total derivative terms [2]. In this paper we will not already omit the total derivative terms. We consider the operator

$$D_i^j = (I g^{\mu\nu} \nabla_\mu \nabla_\nu + \varepsilon K^{\mu\nu} \nabla_\mu \nabla_\nu + W)_i^j, \quad (6)$$

assuming that  $\varepsilon \rightarrow 0$  is a small parameter. We will calculate one-loop divergences up to the terms of the first order in  $\varepsilon$ . Without loss of generality it is possible to consider (as we do) that the matrix  $\varepsilon K^{\mu\nu}_i{}^j$  is symmetric in the indexes  $\mu$  and  $\nu$ . Moreover, we also assume that this matrix and the matrix  $I_i^j$  depend only on the metric tensor  $g_{\alpha\beta}$  (certainly,  $\delta_\alpha^\beta$  and  $g^{\alpha\beta}$  are also possible).

For constructing one-loop divergences we will use the diagram technique. In order to take into account total derivative terms within this technique, we multiply the logarithm of the operator  $D$  by an auxiliary function  $a(x)$  which has not any indexes and calculate a trace of the result:

$$\left( \text{Tr } a(x) \ln D \right)^{(\infty)} = \left( \int d^2 x a(x) \text{tr } \ln D \right)^{(\infty)} = \frac{2}{i} \cdot \frac{1}{4\pi(d-2)} \int d^2 x \sqrt{-g} a(x) b_2, \quad (7)$$

where  $\text{tr}$  denotes the usual matrix trace with respect to the indexes  $i$  and  $j$ . Setting  $a(x) = 1$  we obtain the one-loop contribution to effective action up to the factor  $i/2$ . However, if it is necessary to take carefully into account terms which are integrals of total derivatives, then the presence of a nontrivial function  $a(x)$  is needed. The expression (7) can be presented as a sum of one-loop diagrams in which one of external lines corresponds to the function  $a(x)$ . For this purpose it is convenient to rewrite the operator  $D$  in the following form:

$$D_i^j \equiv I_{0i}{}^j \partial_\mu^2 + \varepsilon K_0^{\mu\nu}{}_i{}^j \partial_\mu \partial_\nu + V_i^j, \quad (8)$$

where the matrices  $I_0$  and  $K_0^{\mu\nu}$  are obtained from the matrices  $I$  and  $K^{\mu\nu}$  by substituting the metric tensor  $g_{\mu\nu}$  (or  $g^{\mu\nu}$ ) by the flat space metric  $\eta_{\mu\nu}$  (or  $\eta^{\mu\nu}$ ). The operator  $V$  includes all other terms. Taking into account the equality

$$\ln(I_0 \partial_\mu^2) = \ln(I_0) + \ln(\partial_\mu^2), \quad (9)$$

diagrams corresponding to the expression (7) can be constructed using the expansion

$$\begin{aligned} \ln(D) &= \ln(I_0 \partial_\mu^2) + \ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right) \\ &+ \frac{1}{2} \left[ \ln(I_0) + \ln(\partial_\mu^2), \ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right) \right] \\ &+ \frac{1}{12} \left[ \ln(I_0) + \ln(\partial_\mu^2), \left[ \ln(I_0) + \ln(\partial_\mu^2), \ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right) \right] \right] \\ &+ \frac{1}{12} \left[ \ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right), \left[ \ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right) \right] \right] \\ &\ln(I_0) + \ln(\partial_\mu^2) \Big] + \dots, \end{aligned} \quad (10)$$

where dots denote terms with a larger number of commutators.

The first term in this expression does not depend on fields. Its contribution to the one-loop divergences is an insignificant constant. The other terms contain

$$\ln \left( 1 + \frac{1}{\partial_\mu^2} I_0^{-1} (\varepsilon K_0^{\mu\nu} \partial_\mu \partial_\nu + V) \right), \quad (11)$$

which can be easily expanded in powers of  $V$  and  $\varepsilon$ . All terms of the zero and the first order in  $V$  or  $\varepsilon$  containing a commutator with  $\ln(I_0)$  vanish after calculating the matrix trace, because the function  $a(x)$  has no matrix indexes.

The remaining terms give the sum of Feynman diagrams in the one-loop approximation. As we already mentioned, one of the external lines in these diagrams corresponds to the function  $a(x)$ . We extract from these diagrams the divergent ones. If the calculations are made on the flat background, then there is the only divergent diagram (1) in Fig. 1.

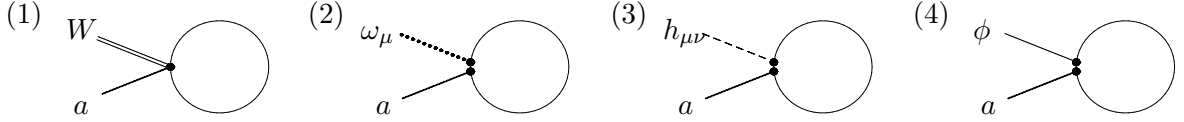


Figure 1: Diagrams in  $d = 2$ , giving the divergent part of the one-loop effective action.

Using Eqs. (7) and (10) we construct analytical expressions for these diagrams, from which we extract logarithmically divergent terms. As usual, they are calculated in the Euclidean space after the Wick rotation. In the considered terms we integrate over angles and, after that, replace the remaining Euclidean integral over the momentum according to the prescription

$$\int \frac{d^d k}{(2\pi)^2 k^2} \rightarrow -\frac{1}{2\pi(d-2)}. \quad (12)$$

In order to generalize the result to the case of the curved space-time it is convenient to use the weak field expansion around the metric of the flat space-time. For this purpose we define

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}, \quad (13)$$

which is a deviation of the metric from the flat metric  $\eta_{\mu\nu}$ . We assume that this expression is small. A number of divergent diagrams in which external lines correspond to  $h_{\mu\nu}$  is infinite. However, a sum of these diagrams is a weak field expansion of a certain covariant result. From dimensional arguments it is easy to see that terms which can appear in the calculation of one-loop divergences should contain the curvature in no more than the first degree. Therefore, from the infinite set of the divergent diagrams it is sufficient to calculate only the diagrams (2) — (4) in Fig. 1. Two small adjacent circles mean that, for example, the derivatives  $\partial_\mu$  can appear between  $h_{\mu\nu}$  and  $a$ .  $\phi^a$  (as in Ref. [2]) denotes the fields  $h_{\alpha\beta}$  which arise from the expansion of  $\varepsilon K^{\mu\nu}{}_i{}^j$  in powers of the deviation of the metric from the flat one. These fields are excluded from the final result by using the identity

$$0 = \nabla_\alpha K^{\mu\nu}{}_i{}^j = \frac{\partial}{\partial \phi^a} K^{\mu\nu}{}_i{}^j \partial_\alpha \phi^a + \Gamma_{\alpha\beta}^\mu K^{\beta\nu}{}_i{}^j + \Gamma_{\alpha\beta}^\nu K^{\mu\beta}{}_i{}^j + \omega_{\alpha i}{}^k K^{\mu\nu}{}_k{}^j - K^{\mu\nu}{}_i{}^k \omega_{\alpha k}{}^j. \quad (14)$$

Calculating the diagrams (1) — (4) according to the algorithm described above and constructing the covariant result using the equations

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2} \left( \partial_\mu \partial_\alpha h_\nu{}^\alpha + \partial_\nu \partial_\alpha h_\mu{}^\alpha - \partial_\mu \partial_\nu h_\alpha{}^\alpha - \partial^2 h_{\mu\nu} \right) + O(h^2); \\ R &= \partial_\mu \partial_\nu h^{\mu\nu} - \partial^2 h_\alpha{}^\alpha + O(h^2); \\ F_{\mu\nu i}{}^j &= \partial_\mu \omega_{\nu i}{}^j - \partial_\nu \omega_{\mu i}{}^j + O(\omega^2), \end{aligned} \quad (15)$$

finally we obtain

$$b_2 = \text{tr} \left( \widehat{W} + \frac{1}{6}R - \frac{1}{2}\varepsilon \widehat{K}_\alpha{}^\alpha \widehat{W} - \frac{1}{12}\varepsilon \widehat{K}_\alpha{}^\alpha R + \frac{1}{6}\varepsilon \widehat{K}^{\mu\nu} R_{\mu\nu} \right), \quad (16)$$

where we use the notation

$$\varepsilon \widehat{K}^{\mu\nu} \equiv I^{-1} \varepsilon K^{\mu\nu}; \quad \widehat{W} \equiv I^{-1} W. \quad (17)$$

The possibility of writing the result in the covariant form can be considered as a nontrivial test of the calculations correctness. Also we note that the result is written in a very compact form (in comparisons with the expression for one-loop divergences in the case of an arbitrary nonminimal operator). Therefore, it is possible to suggest that in the limit  $\varepsilon \rightarrow 0$  the formula for one-loop divergences in  $d = 4$  can be also considerably simplified.

For  $\varepsilon K^{\mu\nu}{}_i{}^j = \varepsilon g^{\mu\nu} \delta_i^j$  and  $I = 1$  the formula (16) correctly reproduces the known result for the  $b_2$  coefficient of the minimal operator (see, for example, [27]) including the total derivative terms.

### 3 One-loop divergences for higher spin fields in the nonminimal gauge

#### 3.1 Higher spin fields on the (A)dS background

Higher spins are described by the totally symmetric tensor fields  $\phi_{\mu_1 \mu_2 \dots \mu_s}$  which satisfy the double tracelessness condition

$$\phi_\alpha{}^\alpha{}_\beta{}^\beta{}_{\mu_3 \dots \mu_s} = 0. \quad (18)$$

(In this paper we will consider the case  $s \geq 3$ .) The free action for these fields on background of the flat space-time has been constructed in Ref. [28]. (Fermion higher spin fields and the action for them are described in Ref. [29].) It is possible to construct an interacting theory for the higher spins [30, 31], but this is a very complicated problem. Even an action quadratic in the higher spin fields on the curved background [32] can be written only if the background geometry is a constant curvature space ((anti) de-Sitter space), for which

$$R_{\mu\nu\alpha\beta} = \frac{1}{d(d-1)}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})R, \quad (19)$$

where  $R = \text{const}$ . It was shown in Ref. [33] that under certain assumptions a consistent Lagrangian formulation for the free boson totally symmetric higher spin fields is possible only in this case. The action for the higher spin fields on the (anti) de-Sitter background is written as

$$\begin{aligned} S = & \frac{(-1)^s}{2} \int d^d x \sqrt{-g} \left[ (\nabla_\alpha \phi_{\mu_1 \dots \mu_s})^2 - \frac{1}{2} s(s-1) (\nabla_\alpha \phi_{\beta}{}^\beta{}_{\mu_3 \dots \mu_s})^2 - s (\nabla^\alpha \phi_{\alpha \mu_2 \dots \mu_s})^2 \right. \\ & + s(s-1) \nabla_\alpha \phi^{\alpha\beta\mu_3 \dots \mu_s} \nabla_\beta \phi_{\gamma}{}^\gamma{}_{\mu_3 \dots \mu_s} - \frac{1}{4} s(s-1)(s-2) (\nabla_\alpha \phi^{\alpha\beta}{}_{\beta \mu_4 \dots \mu_s})^2 + c_1 R (\phi_{\mu_1 \dots \mu_s})^2 \\ & \left. + c_2 R (\phi_{\gamma}{}^\gamma{}_{\mu_3 \dots \mu_s})^2 \right], \end{aligned} \quad (20)$$

where the squares of tensors denote contractions of all free indexes with the metric  $g^{\mu\nu}$ , and the coefficients  $c_1$  and  $c_2$  are

$$c_1 = -\frac{(s-1)(s-4)}{d(d-1)} - \frac{(s-2)}{d}; \quad c_2 = \frac{s(s-1)}{2d} \left( s-1 + \frac{s(s-3)}{d-1} \right). \quad (21)$$

These values are obtained by requiring the gauge invariance of the action under the transformations

$$\delta\phi_{\mu_1 \dots \mu_s} = \frac{1}{s} (\nabla_{\mu_1} \alpha_{\mu_2 \dots \mu_s} + \nabla_{\mu_2} \alpha_{\mu_1 \mu_3 \dots \mu_s} + \dots). \quad (22)$$

Their parameter  $\alpha_{\mu_1 \mu_2 \dots \mu_{s-1}}$  is a totally symmetric tensor which satisfies the tracelessness condition  $\alpha_{\beta}^{\beta}{}_{\mu_3 \dots \mu_{s-1}} = 0$ . In a particular case  $d = 2$ , which is considered in this paper, the coefficients  $c_1$  and  $c_2$  are given by

$$c_1 = -\frac{1}{2}(s^2 - 4s + 2); \quad c_2 = \frac{s(s-1)}{4}(s^2 - 2s - 1). \quad (23)$$

For quantization of gauge theories [34] (and, in particular, the considered higher spin theory on the (anti) de-Sitter background) it is necessary to fix a gauge and add the corresponding Faddeev–Popov ghosts [35]. It is well known that the effective action is gauge independent on shell. However, explicit calculations (see, e.g., Ref. [36]) show that the effective action depends on a gauge off shell. In this paper a gauge is fixed by adding the terms

$$S_{\text{gf}} = \frac{(-1)^s}{2} \int d^d x \sqrt{-g} s(1 + \lambda) \left( \nabla^{\alpha} \phi_{\alpha \mu_1 \dots \mu_{s-1}} - \frac{1}{2}(s-1)(1 + \beta) \nabla_{(\mu_1} \phi^{\alpha}{}_{\alpha \mu_2 \dots \mu_{s-1})} \right. \\ \left. + \frac{\beta(s-1)(s-2)}{2d + 4(s-3)} g_{(\mu_1 \mu_2} \nabla_{\alpha} \phi^{\alpha \beta}{}_{\beta \mu_3 \dots \mu_{s-1})} \right)^2. \quad (24)$$

The last term is added in order that the gauge condition is traceless, and a number of the gauge conditions coincides with a number of gauge transformation parameters. The round brackets denote symmetrization with respect to the indexes  $\mu_1 \mu_2 \dots \mu_{s-1}$ . In the general case,

$$T_{(\mu_1 \mu_2 \dots \mu_k)} \equiv \frac{1}{k!} \left( T_{\mu_1 \mu_2 \dots \mu_k} + T_{\mu_2 \mu_1 \dots \mu_k} + \text{the other permutations of indexes} \right). \quad (25)$$

The Lagrangian for the Faddeev–Popov ghosts is obtained in a standard way by making a gauge transformation in the expression for the gauge condition

$$\nabla^{\alpha} \phi_{\alpha \mu_1 \dots \mu_{s-1}} - \frac{1}{2}(s-1)(1 + \beta) \nabla_{(\mu_1} \phi^{\alpha}{}_{\alpha \mu_2 \dots \mu_{s-1})} \\ + \frac{\beta(s-1)(s-2)}{2d + 4(s-3)} g_{(\mu_1 \mu_2} \nabla_{\alpha} \phi^{\alpha \beta}{}_{\beta \mu_3 \dots \mu_{s-1})}. \quad (26)$$

The gauge parameter becomes the ghost field  $c_{\mu_1 \mu_2 \dots \mu_{s-1}}$ , and the result is multiplied by the antighost field  $\bar{c}^{\mu_1 \mu_2 \dots \mu_{s-1}}$ . As well as the parameters of the gauge transformation, the ghost and antighost fields are totally symmetric and traceless. It is easy to see that the result for the ghost Lagrangian is written as

$$L_{\text{gh}} = \bar{c}^{\mu_1 \mu_2 \dots \mu_{s-1}} \left( \nabla^{\alpha} \nabla_{\alpha} c_{\mu_1 \mu_2 \dots \mu_{s-1}} - \frac{\beta}{2}(s-1)(\nabla_{\mu_1} \nabla^{\alpha} + \nabla^{\alpha} \nabla_{\mu_1}) c_{\alpha \mu_2 \dots \mu_{s-1}} \right. \\ \left. + (\beta + 2) \frac{(s-1)(d+s-3)}{2d(d-1)} R c_{\mu_1 \mu_2 \dots \mu_{s-1}} \right). \quad (27)$$

### 3.2 Calculation of one-loop divergences

In this paper we use Eq. (16) for calculating a divergent part of the one-loop effective action for the considered theory, if  $d = 2$ ,  $s \geq 3$ , and the parameters  $\lambda$  and  $\beta$  are small. In this case in the lowest order in  $\lambda$  and  $\beta$  the sum of the classical action and the gauge fixing term can be written in the form

$$\begin{aligned}
S + S_{\text{gf}} = & \frac{(-1)^s}{2} \int d^2x \sqrt{-g} \left\{ (\nabla_\alpha \phi_{\mu_1 \mu_2 \dots \mu_s})^2 - \frac{1}{4} s(s-1)(1-\lambda-2\beta) (\nabla_\alpha \phi_{\beta \mu_1 \dots \mu_{s-2}})^2 \right. \\
& + \frac{\lambda s}{2} \left( (\nabla^\alpha \phi_{\alpha \mu_1 \dots \mu_{s-1}})^2 + \nabla_\alpha \phi_{\beta \mu_1 \dots \mu_{s-1}} \nabla^\beta \phi^{\alpha \mu_1 \dots \mu_{s-1}} \right) - s(s-1)(\lambda + \beta) \nabla^\alpha \phi_{\alpha \beta \mu_1 \dots \mu_{s-2}} \\
& \times \nabla^\beta \phi_{\gamma}{}^{\gamma \mu_1 \dots \mu_{s-2}} + \frac{1}{8} s(s-1)(s-2)(\lambda + 2\beta) \left( (\nabla_\alpha \phi^{\alpha \beta}{}_{\beta \mu_1 \dots \mu_{s-3}})^2 + \nabla_\gamma \phi^{\alpha \beta}{}_{\beta \mu_1 \dots \mu_{s-3}} \right. \\
& \times \nabla_\alpha \phi_{\delta}{}^{\delta \gamma \mu_1 \dots \mu_{s-3}} \Big) + R(\phi_{\mu_1 \mu_2 \dots \mu_s})^2 \left( -\frac{1}{2}(s^2 - 4s + 2) + \frac{1}{4} \lambda s^2 \right) + \frac{1}{4} R(\phi^\alpha{}_{\alpha \mu_1 \dots \mu_{s-2}})^2 \\
& \left. \times s(s-1) \left( s^2 - 2s - 1 - \lambda - \frac{1}{4}(s-2)^2(2 + \lambda + 2\beta) \right) + o(\lambda, \beta) \right\}, \tag{28}
\end{aligned}$$

where  $o(\lambda, \beta)$  denotes terms of higher orders in  $\lambda$  and  $\beta$ .

The second variation of this expression with respect to the fields  $\phi_{\alpha_1 \alpha_2 \dots \alpha_s}$  is a second order differential operator. Calculating a trace of the logarithm of this operator, we obtain one-loop diagrams with a loop of the spin  $s$  field and external lines corresponding to the field  $h_{\mu\nu}$ . However, it is not necessary to calculate the diagrams in this case, because one can use Eq. (16), which immediately gives the sum of their divergent parts in the covariant form. In order to use this formula, it is necessary to find the second variation of Eq. (28) and, using it, obtain the matrices  $I$ ,  $\varepsilon K^{\mu\nu}$ , and  $W$ . Constructing these matrices it is important to take into account that the fields  $\phi_{\alpha_1 \alpha_2 \dots \alpha_s}$  are double traceless. As a consequence, the projection operators to the double traceless states appear in all matrices. We will denote them by  $Q_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\beta_1 \beta_2 \dots \beta_s}$ . The structure of this projection operator and its properties are discussed in the Appendix.

Having calculated the second variation of the action we find that it is given by a differential operator of the form (6), in which (after omitting an inessential factor)

$$\begin{aligned}
I_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\beta_1 \beta_2 \dots \beta_s} = & Q_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\gamma_1 \gamma_2 \dots \gamma_s} \left( 1_{\gamma_1 \gamma_2 \dots \gamma_s}^{\delta_1 \delta_2 \dots \delta_s} - \frac{s(s-1)}{4} \cdot g_{(\gamma_1 \gamma_2} g^{\delta_1 \delta_2} 1_{\gamma_3 \dots \gamma_s)}^{\delta_3 \dots \delta_s} \right) \\
& \times Q_{\delta_1 \delta_2 \dots \delta_s}{}^{\beta_1 \beta_2 \dots \beta_s}; \tag{29}
\end{aligned}$$

$$\begin{aligned}
\varepsilon K^{\mu\nu}{}_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\beta_1 \beta_2 \dots \beta_s} = & Q_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\gamma_1 \gamma_2 \dots \gamma_s} \left\{ \frac{s(s-1)}{4} (\lambda + 2\beta) g^{\mu\nu} g_{(\gamma_1 \gamma_2} g^{\delta_1 \delta_2} 1_{\gamma_3 \dots \gamma_s)}^{\delta_3 \dots \delta_s} \right. \\
& + \frac{s\lambda}{2} \left( \delta_{(\gamma_1}^\mu g^{\nu(\delta_1} 1_{\gamma_2 \dots \gamma_s)}^{\delta_2 \dots \delta_s)} + \delta_{(\gamma_1}^\nu g^{\mu(\delta_1} 1_{\gamma_2 \dots \gamma_s)}^{\delta_2 \dots \delta_s)} \right) \\
& - \frac{s(s-1)}{2} (\lambda + \beta) \left( \delta_{(\gamma_1}^\mu \delta_{\gamma_2}^\nu g^{\delta_1 \delta_2} 1_{\gamma_3 \dots \gamma_s)}^{\delta_3 \dots \delta_s} + g^{\mu(\delta_1} g^{\nu \delta_2} g_{(\gamma_1 \gamma_2} 1_{\gamma_3 \dots \gamma_s)}^{\delta_3 \dots \delta_s)} \right) \\
& + \frac{s(s-1)(s-2)}{8} (\lambda + 2\beta) \left( \delta_{(\gamma_1}^\mu g^{\nu(\delta_1} g^{\delta_2 \delta_3} g_{\gamma_2 \gamma_3} 1_{\gamma_4 \dots \gamma_s)}^{\delta_4 \dots \delta_s)} + \delta_{(\gamma_1}^\nu g^{\mu(\delta_1} g^{\delta_2 \delta_3} g_{\gamma_2 \gamma_3} 1_{\gamma_4 \dots \gamma_s)}^{\delta_4 \dots \delta_s)} \right) \Big\} \\
& \times Q_{\delta_1 \delta_2 \dots \delta_s}{}^{\beta_1 \beta_2 \dots \beta_s}; \tag{30}
\end{aligned}$$

$$W_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\beta_1 \beta_2 \dots \beta_s} = Q_{\alpha_1 \alpha_2 \dots \alpha_s}{}^{\gamma_1 \gamma_2 \dots \gamma_s} \left\{ R \left( \frac{1}{2}(s^2 - 4s + 2) - \frac{1}{4} \lambda s^2 \right) \cdot 1_{\gamma_1 \gamma_2 \dots \gamma_s}^{\delta_1 \delta_2 \dots \delta_s} \right.$$



$$\begin{aligned}
& -\frac{1}{4}Rs(s-1)\left(s^2-2s-1-\lambda-\frac{1}{4}(s-2)^2(2+\lambda+2\beta)\right) \cdot g_{(\gamma_1\gamma_2)}g^{(\delta_1\delta_2}1_{\gamma_3\ldots\gamma_s)}^{\delta_3\ldots\delta_s}\Big\} \\
& \times Q_{\delta_1\delta_2\ldots\delta_s}{}^{\beta_1\beta_2\ldots\beta_s},
\end{aligned} \tag{31}$$

where we use the notation

$$1_{\gamma_1\gamma_2\ldots\gamma_k}^{\beta_1\beta_2\ldots\beta_k} \equiv \delta_{(\gamma_1}^{\beta_1}\delta_{\gamma_2}^{\beta_2}\ldots\delta_{\gamma_k)}^{\beta_k}. \tag{32}$$

The matrix inverse to  $I$  is defined by

$$I_{\alpha_1\alpha_2\ldots\alpha_s}{}^{\beta_1\beta_2\ldots\beta_s}(I^{-1})_{\beta_1\beta_2\ldots\beta_s}{}^{\gamma_1\gamma_2\ldots\gamma_s} = Q_{\alpha_1\alpha_2\ldots\alpha_s}{}^{\gamma_1\gamma_2\ldots\gamma_s}. \tag{33}$$

From this condition one can obtain an explicit expression for the matrix  $I^{-1}$ , which has the form

$$(I^{-1})_{\alpha_1\alpha_2\ldots\alpha_s}{}^{\beta_1\beta_2\ldots\beta_s} = Q_{\alpha_1\alpha_2\ldots\alpha_s}{}^{\gamma_1\gamma_2\ldots\gamma_s}\left(1_{\gamma_1\gamma_2\ldots\gamma_s}^{\beta_1\beta_2\ldots\beta_s} - \frac{s(s-1)}{4(s-2)} \cdot g_{(\gamma_1\gamma_2)}g^{(\beta_1\beta_2}1_{\gamma_3\ldots\gamma_s)}^{\beta_3\ldots\beta_s}\right). \tag{34}$$

Substituting the matrices (29) — (31) and (34) into Eq. (16) we obtained the following result for the coefficient  $b_2$  corresponding to the second variation of the action (28):

$$b_{2(\text{main})} = \left(2(s-1)^2 - \frac{4}{3} + \beta s(s-1)^2 + o(\lambda, \beta)\right)R. \tag{35}$$

The integral of this expression multiplied by  $\sqrt{-g}$  over  $d^2x$  is proportional to the divergent part of the sum of one-loop Feynman diagrams containing a loop of the spin  $s$  field.

The divergent part of the one-loop effective action is also contributed by diagrams with a loop of the Faddeev–Popov ghosts. This contribution can be also calculated using Eq. (16), which should be applied to the second variation of the ghost action. For  $d = 2$  the corresponding Lagrangian has the form

$$\begin{aligned}
L_{\text{gh}} = & \bar{c}^{\mu_1\mu_2\ldots\mu_{s-1}}\left(\nabla^\alpha\nabla_\alpha c_{\mu_1\mu_2\ldots\mu_{s-1}} - \frac{\beta}{2}(s-1)(\nabla_{\mu_1}\nabla^\alpha + \nabla^\alpha\nabla_{\mu_1})c_{\alpha\mu_2\ldots\mu_{s-1}}\right. \\
& \left. + \frac{1}{4}(\beta+2)(s-1)^2 R c_{\mu_1\mu_2\ldots\mu_{s-1}}\right).
\end{aligned} \tag{36}$$

Taking into account that the ghost fields are totally symmetric and traceless, it is easy to see that the matrices needed for calculations based on Eq. (16) have the following form:

$$\begin{aligned}
\varepsilon K^{\mu\nu}{}_{\alpha_1\ldots\alpha_{s-1}}{}^{\beta_1\ldots\beta_{s-1}} = & -\frac{1}{2}\beta(s-1) \cdot P_{\alpha_1\ldots\alpha_{s-1}}{}^{\gamma_1\ldots\gamma_{s-1}} \\
& \times \left(g^{\mu(\delta_1}\delta_{(\gamma_1}^\nu 1_{\gamma_2\ldots\gamma_{s-1})}^{\delta_2\ldots\delta_{s-1}} + g^{\nu(\delta_1}\delta_{(\gamma_1}^\mu 1_{\gamma_2\ldots\gamma_{s-1})}^{\delta_2\ldots\delta_{s-1}}\right) P_{\delta_1\ldots\delta_{s-1}}{}^{\beta_1\ldots\beta_{s-1}};
\end{aligned} \tag{37}$$

$$W_{\alpha_1\ldots\alpha_{s-1}}{}^{\beta_1\ldots\beta_{s-1}} = \frac{1}{4}(\beta+1)(s-1)^2 R \cdot P_{\alpha_1\ldots\alpha_{s-1}}{}^{\beta_1\ldots\beta_{s-1}}, \tag{38}$$

where  $P_{\alpha_1\ldots\alpha_{s-1}}{}^{\beta_1\ldots\beta_{s-1}}$  is a projection operator to the traceless states in two dimensions. Its structure and also some its properties are described in the Appendix.

Substituting the matrices (37) and (38) into Eq. (16), after some simple transformations we obtain that for diagrams with a loop of the Faddeev–Popov ghosts the expression for  $b_2$  coefficient is

$$b_{2(\text{gh})} = \left((s-1)^2 + \frac{1}{3} + \frac{s(s-1)^2}{2}\beta + o(\beta)\right)R. \tag{39}$$

Let us sum the results for the main contribution and the contribution of the Faddeev–Popov ghosts. Doing this, it is necessary to take into account that the ghost fields are anticommuting that gives the factor  $(-1)$  and differ from antighosts that gives the factor 2. Therefore, the final result for the divergent part of the one-loop effective action for  $s \geq 3$  is written as

$$\Gamma_{1-loop}^{(\infty)} = \frac{1}{4\pi(d-2)} \int d^2x \sqrt{-g} \left( b_{2(\text{main})} - 2b_{2(\text{gh})} \right) = \frac{1}{4\pi(d-2)} \int d^2x \sqrt{-g} \left( -2R + o(\lambda, \beta) \right). \quad (40)$$

This expression does not contain terms of the first order in  $\lambda$  and  $\beta$ . Thus, in the considered approximation the result is gauge independent. Moreover, the result does not depend on a value of  $s$ . For  $s = 3$  all equations obtained here agree with the ones obtained in Ref. [37], in which this particular case has been considered.

## 4 Conclusion

In this paper a simple algorithm for calculating one-loop divergences in two dimensions is proposed in the case when the second variation of the action is a nonminimal operator of the second order, and “nonminimal” terms are small. It is important that the proposed formula allows to calculate terms which are total derivatives. This formula in the considered limit appeared to be very simple. It is manifestly covariant and allows to make calculations easily on the curved space background.

As an application we calculated one-loop divergences for the higher spin theory on the constant curvature background in a nonminimal gauge, which depends on the two parameters  $\lambda$  and  $\beta$ , in the limit in which these parameters are small. By an explicit calculation we demonstrated that in the considered approximation the result is gauge independent. This follows from the fact that the considered effective action is the Green functions generating functional without sources, which does not depend on gauge. Moreover, the calculations showed that the result is independent of the spin value  $s$  for  $s \geq 3$ . Vanishing of the gauge dependence can be also considered as a test of Eq. (16) correctness, especially if one takes into account that all intermediate expressions depend on the gauge parameters in a highly nontrivial way.

Although the case  $d = 2$  is not so interesting as the case  $d = 4$ , the method used in this paper can be applied for making calculations in other dimensions. In the considered limit (when nonminimal terms are small) it is reasonable to expect that the result will be much simpler than the general formula presented in Ref. [2]. Moreover, it becomes possible to take into account total derivatives, which were omitted in Ref. [2].

Possibly, one can also try to find the answer for an arbitrary nonminimal operator for which nonminimal terms are not small taking into account the total derivative terms. However, this problem has not so far been solved.

*Note:* After appearing the first version of this paper on the ArXiv we learned about some results related to the ones obtained in this paper. In particular, the  $b_2$  coefficient for the considered operator has been also calculated in [38] by a different method. We have verified that Eq. (16) is in agreement with this result. Moreover, the independence of the one-loop divergences on  $s$  is possibly related to the cancellation of the vacuum energy in the sum over all spins, regularized by the help of zeta-function [39]. This follows from the triviality of the partition function [40], in which contributions of each spin is given by the ratio of functional determinants [41] that cancel each other in the product.

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## A Appendix: Projection operators to the traceless and double traceless states and their properties.

Since the higher spin fields are double traceless, and the corresponding ghost fields are traceless, expressions for the second variation of the action (or the ghost action) contain projection operators to the double traceless (or traceless) states. In this appendix we describe structure of these projection operators and point out some their properties.

In the explicit form the projection operator to the traceless states in two dimensions is written as

$$P_{\alpha_1 \dots \alpha_{s-1}}^{\beta_1 \dots \beta_{s-1}} = 1_{\alpha_1 \dots \alpha_{s-1}}^{\beta_1 \dots \beta_{s-1}} - x_1 \cdot g_{(\alpha_1 \alpha_2} g^{(\beta_1 \beta_2} 1_{\alpha_3 \dots \alpha_{s-1}}^{\beta_3 \dots \beta_{s-1}}) - x_2 \cdot g_{(\alpha_1 \alpha_2} g_{\alpha_3 \alpha_4} g^{(\beta_1 \beta_2} g^{\beta_3 \beta_4} 1_{\alpha_5 \dots \alpha_{s-1}}^{\beta_5 \dots \beta_{s-1}}) - \dots, \quad (41)$$

where  $x_1, x_2, \dots$  are numerical coefficients depending on  $s$  and  $d$ . In this paper we need not explicit expressions for these coefficients. They can be found, e.g., in Ref. [20]. The traceless projection operator has the following properties:

$$\begin{aligned} P_{\alpha_1 \alpha_2 \dots \alpha_{s-1}}^{\beta_1 \beta_2 \dots \beta_{s-1}} g^{\alpha_1 \alpha_2} &= 0; & P_{\alpha_1 \alpha_2 \dots \alpha_{s-1}}^{\beta_1 \beta_2 \dots \beta_{s-1}} g_{\beta_1 \beta_2} &= 0; \\ P_{\alpha_1 \alpha_2 \dots \alpha_{s-1}}^{\beta_1 \beta_2 \dots \beta_{s-1}} P_{\beta_1 \beta_2 \dots \beta_{s-1}}^{\gamma_1 \gamma_2 \dots \gamma_{s-1}} &= P_{\alpha_1 \alpha_2 \dots \alpha_{s-1}}^{\gamma_1 \gamma_2 \dots \gamma_{s-1}}. \end{aligned} \quad (42)$$

Two first equalities are actually a part of the projection operator definition, and the last one is their straightforward consequence. Moreover, in the case  $d = 2$  and  $s \geq 2$

$$\text{tr } P = P_{\alpha_1 \dots \alpha_{s-1}}^{\alpha_1 \dots \alpha_{s-1}} = 2. \quad (43)$$

This can be easily verified by calculating a number of independent components for the traceless field in two dimensions.

Similarly, the projection operator to the double traceless states has the form

$$\begin{aligned} Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} &= 1_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} - y_1 \cdot g_{(\alpha_1 \alpha_2} g_{\alpha_3 \alpha_4} g^{(\beta_1 \beta_2} g^{\beta_3 \beta_4} 1_{\alpha_5 \dots \alpha_s}^{\beta_5 \dots \beta_s)} \\ &- y_2 \cdot g_{(\alpha_1 \alpha_2} g_{\alpha_3 \alpha_4} g_{\alpha_5 \alpha_6} g^{(\beta_1 \beta_2} g^{\beta_3 \beta_4} g^{\beta_5 \beta_6} 1_{\alpha_7 \dots \alpha_s}^{\beta_7 \dots \beta_s)} - \dots, \end{aligned} \quad (44)$$

where  $y_1, y_2, \dots$  are numerical coefficients depending on  $s$  and  $d$ , which satisfy the following properties:

$$\begin{aligned} Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} g^{\alpha_1 \alpha_2} g^{\alpha_3 \alpha_4} &= 0; & Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} g_{\beta_1 \beta_2} g_{\beta_3 \beta_4} &= 0; \\ Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} Q_{\beta_1 \beta_2 \dots \beta_s}^{\gamma_1 \gamma_2 \dots \gamma_s} &= Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\gamma_1 \gamma_2 \dots \gamma_s}. \end{aligned} \quad (45)$$

In the case  $d = 2$  the following identities are also valid:

$$\begin{aligned} \text{tr } Q &= Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\alpha_1 \alpha_2 \dots \alpha_s} = 4, \quad \text{if } s \geq 3; \\ Q_{\alpha_1 \alpha_2 \dots \alpha_s}^{\beta_1 \beta_2 \dots \beta_s} g^{\alpha_1 \alpha_2} g_{\beta_1 \beta_2} &= \frac{4}{s} \cdot P_{\alpha_3 \dots \alpha_s}^{\beta_3 \dots \beta_s}. \end{aligned} \quad (46)$$

The trace of the projection operator  $Q$  can be found by calculating a number of independent components for the double traceless field in two dimensions. In order to verify the last identity we note that the left hand side is evidently proportional to the traceless projection operator, and the coefficient can be found by comparing terms proportional to  $1_{\alpha_3 \dots \alpha_s}^{\beta_3 \dots \beta_s}$ .

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